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SOME CONFORMALLY FLAT NON – STATIC CHARGED FLUID SPHERE IN EINSTEIN – CARTAN – MAXWELL THEORY

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Abstract:

The present paper provides some solution for non – static conformally flat charged fluid spheres in Einstein - Cartan – Maxwell Theory in different cases. Here we have also found and discussed various physical and geometrical features of the models e.g. pressure, density, expansion rotation shear and field tensor F_{14}. It is seen that model (when F_{14}=0) is expanding, rotating but non –geodetic in general. Further in comoving co – ordinates and $F_{14} \neq$ 0, then we get the model to be expanding with time but non – rotating and non – shearing.

Key Words: Conformally flat, expansion, shear, rotation, spin, pressure e.t.c.

1. Introduction:

In recent years there has been shown a lot of interest in obtaining solutions for non – static conformally flat charged perfect fluid distribution with spherical symmetry in Einstein Cartan – Maxwell theory. Singh and Abdussattar [16] have obtained a non - static generalization of the Schwarzchild interior solution which is conformal to flat space – time. Chang [1] obtained some conformally flat interior solutions of the Einstein – Maxwell equations for a charged static sphere which is stable and these fulfill physical condition inside the sphere. Gurses [5] has shown that the Schwarzchild interior metric is the only conformally flat static solution of the Einstein field equations with perfect fluid distribution. Roy and Raj Bali [14] have found the solutions of Einstein's field equations giving non – static spherically symmetric perfect fluid distribution which is conformally flat.Prassanna [12] has described the Einstein - Cartan equations with special reference to a perfect fluid distribution following the work of Trautman and has found three solutions taking Hehl's [7, 8] view and Tolman's [17] method . He has dicussed that a space –time metric similar to the interior Schwarzchild solution will no longer show a homogenous fluid sphere in the presence of spin density. Recently Kallyanshetti and Waghmode [9] taking the static conformally flat perfect fluid –disribution with spherical symmetry and obtained the field equations of Einstein – Cartan theory. They found. Some other reseachers in this field are Yadv et al. [18 ,19], Pandya et al. [13] Hansraj [6], Chaisi and Maharaja [2] , Dubey and Singh [3] , et al. Shee-D et al [15] and Manjonjo et al.[10 , 11]. In this paper we have studied the non – static spherically symmetric charged perfect fluid distribution with conformal flatness in Einstein –Cartan –Maxwell theory and have obtained solution of field equations in different cases. Theexplicit expression for pressure, density, expansion, rotation, shear and non – vanishing components of flow vector u_i and field tensor F_{14} have also been obtained. It is seen that model (when F_{14}=0) is expanding , rotating , shearing but non – geodetic in general .However , if we use commoving co- ordinates and $F_{14} \neq 0$ then we find that model representing the perfect fluid distribution is expanding with time but non – shearing .

2. The Field Equations:

We consider a non – static conformally flat spherically symmetric perfect fluid distribution represented by the space – time metric.

(2.1) $ds^2 = e^{\lambda}(-dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + dt^2)$

Where λ is a function of r and t alone.

The Cartan's equations are

(2.2) $Q^i_{jk} - \delta^i_j Q^l_{lk} - \delta^i_k Q^l_{jl} = -kS^i_{jk}$

Where Q^i_{jk} and S^i_{jk} are the torsion tensor and spin tensors respectively.

Also we have

(2.3) $S^i_{jk} = u^i S_{jk}$

with $S_{jk} u^k = 0$

where u^i is four velocity vector.

The only non – vanishing components of the spin S_{jk} is $S_{23} = K$ (say).

From the non – static condition , we have the velocity for vector

$u^i = \delta^i_j$, I , j = 1, 4

Thus the non –zero components of S^i_{jk} are

(2.4) $S^1_{23} = -S^1_{32} = u^i S_{23} = 1$, K=K

Therefore from Cartan's equations the non – zero components of Q^i_{jk}

(2.5) $Q^1_{23} = -kK$ and $Q^4_{23} = -kK$

For charged fluid the Einstein – Cartan –Maxwell equations are

(2.6) $R_{ij} - \frac{1}{2} R g_{ij} + \wedge g_{ij} = -8\pi \overline{T_{ij}}$

(2.7) $Q^i_{jk} - \delta^i_j Q^{\ell}_{\ell k} - \delta^i_k Q^{\ell}_{j\ell} = -kS^i_{jk}$

(2.8) $\left[(-g)^{\frac{1}{2}} F^{ij} \right]$, $j = 4\pi J^{i} (-g)^{\frac{1}{2}}$

(2.9) $F_{[ij,k]} = 0$

where $\wedge$ is a cosmological constant , $\overline{T_j^i}$ is the energy momentum tensor , Q^i_{jk} is torsion tensor , S^i_{jk} is spin tensor , J^i is the current four vector R_{ij} is the Ricci tensor and R the scalar of curvature tensor , k = 8 π .

For the system under study the energy momentum tensor $\overline{T_j^i}$ has two parts viz . T^i_j and E^i_j for matter and charges respectively i.e.

$$\overline{E^i_j} = T^i_j + E^i_j$$

(2.10) $T^i_j = \left[(\rho + p) u^i u_j - p\delta^i_j \right]$

With

(2.11) $g_{ij} u^i u^j = -1$

where p and ρ are pressure and density and u_i = (u_1 , 0, 0, u_4) is the flow vector which describes the radiational motion of the fluid.

The electromagnetic enerfy momentum tensor E^i_j in terms of field tensor F_{ij} is given by

(2.12) $E^i_j = -F_{j\mu} F^{i\mu} + \frac{1}{4} \delta^i_j F_{\mu\upsilon} F^{\mu\upsilon}$

Due to spherical symmetry the only non – vanishing components of field tensor F^{ij} are F^{14}

The above field equations for our metric are

(2.13) $\frac{3\lambda_1^2}{4} + \frac{2\lambda_1}{r} - \lambda_{44} - \frac{\lambda_4^2}{4} - \wedge e^{\lambda} + \frac{1}{2} k^2 K^2 = 8\pi \left[(\rho + p) u_1^2 + pe^{\lambda} - e^{-\lambda} (F_{14})^2 \right]$

(2.14) $\lambda_{11} + \frac{\lambda_1^2}{4} + \frac{\lambda}{r} - \lambda_{44} - \frac{\lambda_4^2}{4} - \wedge e^{\lambda} = 8\pi \left\{ pe^{\lambda} + e^{\lambda} (F_{14})^2 \right\}$

(2.15) $-\lambda_{44} - \frac{\lambda_4^2}{4} - \frac{2\lambda_1}{r} + \frac{3\lambda_4}{r} + e^{\lambda} + \frac{1}{n} k^2 K^2 e^{\lambda} = 8\pi \left[(\rho + p) u_4^2 - pe^{\lambda} + e^{-\lambda} (F_{14})^2 \right]$

(2.16) $-\lambda_{14}+\frac{\lambda_1\lambda_4}{4}-\frac{1}{2}k^2K^2e^{\lambda}=8\pi(\rho+p)u_1u_4$

Equation (2.7) yields

(2.17) $u_4^2-u_1^2=e^{\lambda} \quad \Rightarrow u_1^2=u_4^2-e^{\lambda}$

In the above the suffices 1 and 4 denote differentiation with respect to tr and t respectively.

From equations (2.13) and (2.14)

(2.18) $8\pi\left[(\rho+p)u_1^2-2e^{\lambda}(F_{14})\right]=\frac{\lambda_1^2}{2}+\frac{\lambda_1}{r}-\lambda_{11}+\frac{1}{2}k^2K^2e^{\lambda}$

From equations (2.11) and (2.12) , we have

(2.19) $8\pi\left[(\rho+p)u_4^2+2e^{\lambda}(F_{14}^2)\right]=\frac{\lambda_4^2}{2}-\frac{\lambda_1}{r}-\lambda_{44}+\frac{1}{2}k^2K^2e^{\lambda}$

Equations (2.17) ,(2.18) and (2.19) lead to

(2.20) $8\pi\left[(\rho+p)u_4^2+4e^{-\lambda}(F_{14}^2)\right]=\frac{\lambda_4^2}{2}-\lambda_{44}-\frac{\lambda_1^2}{2}+\lambda_{11}-\frac{2\lambda_1}{r}$

From equations (2.14) and (2.20) , we have

(2.21) $8\pi\left[(\rho+3(F_{14})^2\right]=e^{-\lambda}\left[\frac{3\lambda_4^2}{4}-\frac{3\lambda_1^2}{4}-\frac{3\lambda_1}{r}\right]+\wedge$

Following Hehl's [4,5] approach by redefining pressure and density as

(2.22) $\overline{p}=p-2\pi K^2, \qquad \overline{\rho}=\rho-2\pi K^2$

Equations (2.16) , (2.18) and (2.19) can be written as

(2.23) $8\pi(\overline{\rho}+\overline{p})u_1u_4=-\lambda_{14}+\frac{\lambda_1\lambda_4}{2}$

(2.24) $8\pi(\overline{\rho}+\overline{p})u_1^2=\frac{\lambda_1^2}{n}+\frac{\lambda_1}{r}-\lambda_{11}-2e^{\lambda}(F_{14})^2$

(2.25) $8\pi(\overline{\rho}+\overline{p})u_4^2=\frac{\lambda_4^2}{2}-\frac{\lambda_1}{r}-\lambda_{44}-2e^{\lambda}(F_{14})^2$

3. Solution of the Field Equations :

We try to find the solution i.e. construct the model in the following different cases.

(i) $F_{14}\neq 0, u_1=0$ (i.e. commoving co –ordinates)

(ii) $F_{14}=0, u_1 \neq 0$

(iii) $F_{14}=0, \lambda_{14}=0, u_1 \neq 0$

A. Model I : Here we taken $u_1=0$, $F_{14} \neq 0$

In equation (2.24)if we put $u_1=0$, we get

(3.1) $$2e^{\lambda}(F^{14})^2=\frac{\lambda_1^2}{4}+\frac{\lambda_1}{2r}-\frac{\lambda_{11}}{2}$$

From equation (2.23) we have

(3.2) $$2\lambda_{14}-\lambda_1\lambda_4=0$$

which on integration yields

(3.3) $$e^{\lambda}=\left[\alpha(r)+\beta(t)\right]^{-2}$$

where α and β are functions of r and t respectively . Hence the line element is given by

(3.4) $$ds^2=\frac{1}{(\alpha+\beta)^2}(dr^2+r^2d\theta^2r^2\sin^2\theta d\phi^2-dt^2)$$

SOME PHYSICAL AND GEOMETRICAL FEATURES

(3.5) $$8\pi p=16\pi^2k^2+3(\alpha_1^2+\beta_4^2)+(\alpha+\beta)\left(2\beta_{44}-\alpha_{11}-\frac{3\alpha_1}{r}\right)$$

(3.6) $$8\pi\rho=3(\beta_4^2-\alpha_1^2)+3(\alpha+\beta)\left(\alpha_{11}+\frac{\alpha_1}{r}\right)+16\pi^2K^2$$

The non – zero components of the flow vector u_4 is given by

(3.7) $$u_4=(\alpha+\beta)^{-1}$$

The reality conditions (Ellis[5]) given by

(3.8) $\rho+p>0$

(3.9) $\rho+3p>0$

Give

(3.10) $$\beta_{44}+\alpha_{11}+\frac{\alpha_1}{r}+32\pi^2k^2>0$$

(3.11) $$\left(\alpha_1^2-\beta_4^2\right)+(\alpha+\beta)\left(\beta_{44}-\frac{\alpha_1}{r}\right)+64\pi^2k^2>0$$

F_{14} is given by

(3.12) $$F_{14}=(\alpha+\beta)^{\frac{-3}{4}}\left(\frac{\alpha_1}{r}-\alpha_{11}\right)^{\frac{1}{2}}$$

The expansion Θ, rotation w_{ij} are shear σ_{ij} are givn by

(3.13) $\Theta = u_{ij}$

(3.14) $w_{ij} = u_{ij} - u_{ji}$

(3.15) $\sigma_{ij} = u_{ij} - u_{ji} - \frac{1}{3}\Theta h_{ij}$

where $h_{ij} = (g_{ij} - u_i u_j)$ are found to be

(3.16) $\Theta = 3\beta_4$

(3.17) $w_{ij} = 0$

(3.18) $\sigma_{ij} = 0$

Hence we see that model (3.4) giving the distribution of charged perfect fluid is expanding with time but non – rotating and non – shearing.

4. MODEL II :

Here we take $F_{14} = 0$, $u_1 \neq 0$

Then from equations (2.23) , (2.24) and (2.25) , we have

(4.1) $$\lambda_{11}\left(\frac{\lambda_1}{r} - \frac{\lambda_4^2}{2}\right) + \lambda_{14}\lambda_1\lambda_4 + \lambda_{44}\left(\frac{-\lambda_1^2}{2} - \frac{\lambda_1}{r}\right) + \left(r_{11}r_{44} - \lambda_{14}^2\right) = \frac{\lambda_1}{2r}\left(\lambda_1^2 - \lambda_4^2 + \frac{2\lambda_1}{r}\right)$$

Solution of (4.1) using Monge's method is found to be

(4.2) $$e^{\lambda} = 16A^2\left[f\left(A\left(r^2 - t^2\right) - t\right) - Bt\right]^{-2}$$

where Aand B are constants.

Hence the metric (2.1) takes the forms

(4.3) $$ds^2 = 16A^2\left[f\left(A\left(r^2 - t^2\right) - t\right) - Bt\right]^{-2}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - dt^2\right)$$

The solution obtained by Singh and Abdussattar [16] is a particular case of (4.3) with B=0. Also when B=0 , the metric (4.3) transforms to Robertson Walker metric of constant negative curvature .

The pressure and density for the model (4.3) are given by

(4.4)

$$8\pi P = \left(\frac{1 - 4A\zeta}{16A^2}\right)\left[2 f''\left(f - Bt\right) - 3 f'^2\right] - \frac{1}{16A^2}\left[12Af'\left(f - Bt\right) + 6B\left(2At + 1\right) + 3B^2 - \wedge + 16\pi^2 k^2\right]$$

And

(4.5) $8\pi\rho = \frac{1}{16A^2}\left[3(1-4A\zeta)f'^2 + 6B(2At+1)f' + \frac{3f'(f-Bt)}{4A}\right] + \wedge + 16\pi^2 k^2$

where $\zeta = A(r^2 - t^2) - t$ and a prime denotes differentiation with respect to its argument.

The non – vanishing parts of flow vector are u_1 and u_4 given by

(4.6) $u_1 = \frac{8\pi A^2 r}{(r-Bt)\sqrt{1-4A\zeta}}$

(4.7) $u_4 = \frac{4A(2At+1)}{(f-Bt)\sqrt{1-4A\zeta}}$

The reality conditions (Ellis , 1971) give

(4.8) $K^2 > \frac{(Bt-f)(1-4A\zeta)}{256\pi^2}$ also the second reality condition (3.9) gives

(4.9)

$$\wedge < \frac{3(1-4A\zeta)}{10A^2}\left[f''(f-Bt) - f'^2\right]\frac{3}{8A^2}\left[2Af'(f-Bt) + B(2At+1)f'\right]\frac{3B^2}{16A^2} + 32\pi^2 k^2$$

The expression for expansion θ, rotation w_{ij} and shear σ_{ij} are given by

(4.10) $\theta = \frac{3}{4A\sqrt{1-4A\zeta}}\left[2A(f-Bt) + f'(1-4A\zeta) - B(2At+1)\right]$

(4.11) $w_{14} = \frac{4A^2 rB}{(f-Bt)^2\sqrt{1-4A\zeta}}$

(4.12(a)) $\sigma_{11} = \frac{8AB(2At+1)(1-4A\zeta+4A^2r^2)}{(f-Bt)^2(1-4A\zeta)^{\frac{3}{2}}}$

(4.12(b)) $\sigma_{22} = \sin^2\theta\sigma_{33} = \frac{8ABr^2(2At+1)}{(f-Bt)^2\sqrt{1-4A\zeta}}$

(4.12(c)) $\sigma_{44} = \frac{4AB(2At+1)\left\{2(2At+1)^2 - (1-4A\zeta)\right\}}{(f-Bt)^2(1-4A\zeta)^{\frac{3}{2}}}$

(4.12(d)) $\sigma_{14} = \frac{4A^2Br\left\{4(2At+1)^2 + 1 - 4A\zeta\right\}}{(f-Bt)^2(1-4A\zeta)^{\frac{3}{2}}}$

The flow vector u_i does not satisfy $u_{i,j}\ u^j = 0$ in general and threrfore flow is non – geodetic in general. Sothe model is expanding , rotating , shearing but non – geodetic in general.

5. MODEL III

In this case equation (4.1) reduces to

(5.1) $$\lambda_{11}\left[\frac{\lambda_1}{r}-\frac{\lambda_4^2}{2}\right]+\lambda_{44}-\left(\frac{\lambda_1^2}{2}-\frac{\lambda_1}{r}\right)+\lambda_{11}\lambda_{44}=\frac{\lambda_1}{2r}\left(\lambda_1^2-\lambda_4^2+\frac{2\lambda_1}{r}\right)$$

Integration of $\lambda_{14}=0$ provides us

(5.2) $$\lambda=\phi(r)+\psi(t)$$

By finding $\phi(r)$ and $\psi(t)$, λ can be found and thus metric can be written from (2.1).

6. CONCLUSION AND DISCUSSION:

In this paper taking conformally flat non – static spherically symmetric metric , we have solved Einstein – Cartan - Maxwell field equations in different cases e.g. by taking $u_1=0$ i.e. in co-moving co-ordinates and also $F_{14}\neq 0$ while in other case ,we have taken $u_1\neq 0$ but $F_{14}=0$.We have found different physical and geometrical features for the models we have constructed , we have shown that model **I** (3.4) for charged perfect fluid distribution is expanding with time but non- rotating and non – shearing . Model **II** givenby (4.3) provides results of Singh and Abdussattar [16] when B=0.Also when B=0, the metric (4.3) transforms to Robertson – Walker metric of constant negative curvature. Also in model **II,** since the flow vector does not satisfy $u_{i,j}u^j=0$ in general and hence flow is non – geodesic. So, this model is expanding, rotating, shearing but non – geodetic in general. Our study made here is very useful for astrophysics and astronomical investigation.

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